This deal from the Surfers Teams has a sting in the tail. South plays 6NT after an auction 1D-2C <> 2NT-6NT. The 2NT bid showed 12-14 and, in our system, denied a 4-card major or a 6-card diamond suit. West leads the ♥8. East’s A wins and he returns the ♥4 to dummy. West follows with the 9 (showing, with their carding, either 3 or 4 hearts).

Declarer cannot succeed unless the diamond finesse works, so he leads the ♦4 to his Q and is relieved when this holds; both defenders follow suit.

How should declarer proceed?

Importantly, what is the sting in the tail that I allude to?

Solution:

It appears that West was dealt either ♥982, ♥983 or ♥9832 because the ♥4 return would be East’s original 4th highest.

Declarer now goes back to dummy with a spade and repeats the ♦ finesse. It will win (West isn’t going to decline the setting trick on the first finesse). If East was dealt K doubleton, claim. If not, play the ♦A. If the diamonds are 3-3, claim; we only need 3 club tricks and so bad club breaks are not an issue. (An advantage in playing diamonds first is the clear knowledge of how many club tricks are needed.)

Given the information available after trick 3, there is a 61.81% chance that East has 2-3 diamonds. On the remaining 38.19%, the deal is still alive. We now know that only 4 club tricks are needed. Declarer plays his major-suit winners ending in the South hand. This reduces his holding to:

North: ♠- ♥- ♦- ♣AK97
South: ♠- ♥- ♦10 ♣Q82

In the process, he learns quite a bit about his opponents’ distribution. South observes who follows suit on the major leads and what West discards on the diamonds.

East showing out on the 1st or 2nd round of spades:

If East shows out on the 1st round of spades (0.92% of alive hands1), he will have:

- either 0-5-5-3 or 0-4-5-4 if he is known to have 5 diamonds
- or 0-5-4-4 or 0-4-4-5 if known to have 4 diamonds.

In a number of these cases, East must pitch clubs to hold his ♦K guard. In any case, South knows he can play the top 3 clubs to set up the ♦9 as boss.

If East shows out on the 2nd round of spades (11.07%), he will have 1-5-5-2 or 1-4-5-3 if known to hold 5 diamonds – no problems for declarer here. If known to hold 4 diamonds, he was dealt either 1-4-4-4 or 1-5-4-3. Again, South comes home by playing the top 3 clubs.

East showing out on the 3rd round of spades:

If East shows out on the 3rd round of spades (36.78%), he will have:

- 2-5-5-1 or 2-4-5-2 (when known to hold 5 diamonds) or
- 2-5-4-2 or 2-4-4-3 (if known to have 4)

and declarer will know which. In the latter case, declarer will always succeed (and know that he will succeed) by playing the top 3 clubs. In the former, let us switch attention to West, who has either 5-3-1-4 or 5-4-1-3, and consider the plays she might try.

Firstly, West knows a great deal about South’s hand from the bidding and the play. If West herself does not hold the ♦J, the contract is cold if South has it (along with the two black queens he is marked with by his point-count). So, at an early stage, she assumes that South has 12 HCP comprising ♦AQJ, ♠QJ and ♣Q and that her partner, East, has the ♦J.

Likewise, should West have ♦Jxxx, she sees that South will make the contract if holding ♦Q10x (by the unblocking play of the 10 to the A) and fail in the endgame if holding doubletons ♦Q10 or ♦Qx. So West assumes that her partner has the ♦10.

South’s shape will also be known, because this is a situation where East will give count on

1 All percentages quoted now refer to deals which are still ‘alive’, that is, East has >3 diamonds.
diamonds knowing that this information cannot help declarer, only his partner.

True, South might be 3-3-5-2, but then a West with 4 clubs knows that declarer is going down unless he holds both ♠Q and ♠J.

So West, if she holds 4 clubs, assumes that the end game will be as shown in my earlier diagram -- and she knows her own club holding, either J10xx, Jxxx or 10xxx.

If you were South, how would you play the club suit if you knew West had 4 clubs and East one? You might reason (correctly) that West will have J10xx on 60% of occasions, with Jxxx and 10xxx accounting for 20% each. So you might play low to the ♠9, with the proviso that, should West split her honours, you win, return to the Q and repeat the finesse.

How would you play, however, if you knew that West had the precise 5-3-1-4 distribution because she opted to show you this by her choice of discards on the diamonds? (She can reveal this shape by electing to pitch a heart and a spade, whereas with 5-3-1-4 she could keep you guessing as to 5-3-1-4 or 5-4-1-3 by pitching two spades. With 5-4-1-3, she certainly will pitch two spades.) Is West trying to persuade you to play low to the ♠9 when she holds Jxxx or 10xxx?

Rest assured, that East will cooperate in West’s “poker-like bluffing ploy” by not discarding his 5th heart, should he have 5 -- because East can read South’s hand perfectly too.

This is a neat little game between West and South, in the spirit of “the mathematical theory of games” established during the 1930s by John von Neumann. Often with such games, the participant’s optimal strategy is a randomized one, so we must ask “should West be randomizing here when she holds 4 clubs and should South be countering with a randomized decision between “drop” and “low to 9”?

I have analysed this according to the game-theoretic principles of von Neumann. The optimal strategy for West, at von Neumann’s game “equilibrium”, is:

• always discard two spades holding a 5-4-1-3 shape;
• holding 5-3-1-4, either always pitch at least one heart with ♠Jxxx or ♠10xxx and always pitch two spades with ♠J10xx or pitch at least one heart from J10xx with a probability which is 1/3 the combined probability that you will pitch at least one heart when holding Jxxx or 10xxx [ie. 3p(J10xx) = p(Jxxx)+p(10xxx)].

The second of these options is somewhat esoteric; the first is more practical. Yet both options, and any arbitrary mix of the two, exist in the von Neumann equilibrium. For South, things are decidedly practical. The optimal strategy is to ignore West’s bluffs and simply play for the drop every time. Basically, although West might try to bluff holding 5-3-1-4 to induce a wrong guess from South, declarer will always play for the drop because that line always wins when West is 5-4-1-3 and picks up enough of the 5-3-1-4 cases to be able to secure an overall 70% of the cases inherent in this small “sub-game”.

**East following to 3 round of spades:**
Other interesting “sub-games” between West and South take place when East follows to all 3 rounds of spades (50.60% of cases where the ♦K doesn’t fall).

If this occurs, West is likely to have club length. Indeed, given that she has <5 spades and 2 diamonds, her shape is one of the following:

<p>| | | | |</p>
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<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>2-4-2-5</td>
<td>3.25%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3-3-2-5</td>
<td>7.25%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3-4-2-4</td>
<td>18.16%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4-3-2-4</td>
<td>36.32%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4-4-2-3</td>
<td>36.32%</td>
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</tbody>
</table>

Table D2

Given that she has <5 spades and 1 diamond, her possible shapes are:

<p>| | | |</p>
<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>3-4-1-5</td>
<td>12.5%</td>
<td></td>
</tr>
<tr>
<td>4-3-1-5</td>
<td>25.0%</td>
<td></td>
</tr>
<tr>
<td>4-4-1-4</td>
<td>62.5%</td>
<td></td>
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</tbody>
</table>

Table D1

Consider Table D2 first. There are different discarding ploys available for each of these West holdings when unable to follow to the 3rd diamond.

• With 2-4-2-5, West is doomed, as no discard stops South learning of the precise shape.
• With 3-3-2-5, West should pitch a club to avoid showing out on a major. She might confuse the picture, particularly if she (seemingly irrationally) also pitches a club on some of the other holdings in Table D2.
• With 3-4-2-4, West must choose, possibly randomly, between:
* a ♠, thereby telling South her precise shape;
* a ♥, keeping South unsure between 3-4-2-4 and 4-4-2-3;
* a ♣, but only if she doesn’t hold J10xx, creating a bluff that she has 5 clubs.
• Similarly with 4-3-2-4, West chooses, possibly randomly, between:
  * a ♥, telling South her precise shape;
  * a ♠, keeping South guessing between 4-3-2-4 and 4-4-2-3;
  * a ♣ when she doesn’t hold J10xx, making the same bluff.
• Holding 4-4-2-3, her pitch could be:
  * a ♠, which will say that she is either 4-4-2-3 or 4-3-2-4;
  * a ♥, telling South that she has either 4-4-2-3 or 3-4-2-4;
  * a “bluffing” ♣ from Jxx, 10xx or xxx.

These club bluffs may induce South to finesse into East’s club honour. South must sort out this deceptive minefield.

Many hours of analysis later, it turns out that the D2-game equilibrium results in South succeeding with a chance 83/138. He can guarantee this proportion of success by, and this may surprise (and certainly relieve) the reader, always playing for the drop in clubs, except when West is marked for 2-4-2-5.

West can guarantee that she sets the contract 55/138 of the time by the following strange strategy:
• concede with 2-4-2-5;
• always pitch a club from 3-3-2-5;
• pitch a spade with probability 2/3, otherwise a heart, from the 3-4-2-4 holding that has ♠J10xx;
• always pitch a spade when holding 4 clubs in other contexts;
• with 4-4-2-3, pitch a spade except when holding ♠Jxx, whereby a heart is chosen 2/3 of the time and a club 1/3.

This is not the only “equilibrium” strategy for West, but it suffices to establish the game’s solution.

Now consider Table D1, where West must make two discards. Similar bluffs using a club pitch first are available to West, but these have no effect on declarer, it turns out. South does best to ignore West’s behaviour and always play low to the ♠9, guaranteeing himself success with chance 3/4. West guarantees herself at least a 1/4 success rate by:
• pitching a ♠ then a ♥ from 3-4-1-5;
• pitching a ♠ then a ♠ from 4-3-1-5;
• always discarding a ♥ and a ♠ from 4-4-1-4.

A bluff with clubs here, holding Jxxx or 10xxx, is unnecessary as South will never go for the drop anyway.

**The final percentage:**
South succeeds with a chance of 91.46%, this figure encapsulating the fact that he wins 77.65% of the deals which are still alive after the play of the diamond suit. This is a few percent higher than the line of “playing clubs earlier, without cashing all the major-suit winners”.